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Theoretical Calculation of the Strain-Hardening Exponent and the Strength Coefficient of Metallic Materials

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The purpose of the present article is to theoretically calculate the strain-hardening exponent and the strength coefficient of metallic materials. For this purpose, two equations are used. The first one correlates the strain-hardening exponent and the strength coefficient with the yield stress-strain behavior, while the other one correlates the fracture strength and the fracture ductility. From these two equations, the expressions of both the strain-hardening exponent and the strength coefficient are deduced. Theoretical results from the deduced expressions are then compared with test data. Through the comparison of equations and data, if adequate test data are lacking, the deduced expressions can be used to theoretically calculate the strain-hardening exponent and the strength coefficient for metallic materials. The characteristics of the theoretical approach are simple and easy to use. In addition, the theoretical results can be further applied to examine the correctness of the test data.

Keywords fracture ductility, fracture strength, strain-hardening exponent, strength coefficient, yield stress-strain

1. Introduction

It is well known that both the strain-hardening exponent and the strength coefficient are basic mechanical behavior performance parameters of metallic materials. When the tensile properties of metallic materials are being evaluated, these two parameters must be known. Also, when the fatigue crack-initiation lifetime of a loaded structural component using the equivalent stress amplitude method are being studied (Ref 1-3), these two performance parameters must be known. Even though these performance parameters can be determined experimentally, they are often calculated theoretically because comprehensive test data are not usually available. Traditionally, there exist two equations (Ref 1-6) used to theoretically calculate the strain-hardening exponent and the strength coefficient: One equation is used to correlate the strain-hardening exponent and the strength coefficient with the yield stress-strain behavior. The other equation correlates the two performance parameters with the fracture strength and the fracture ductility. The basis of the two equations is found in the Hollomon equation (Ref 7). However, as well known as the Hollomon equation is (i.e., a fitted equation using tensile stress-strain test data points), when the equation is used at specific points deviation problems may arise. Therefore, the correctness and the precision of the two equations have been examined in this work. Previously, the applicability of using these two equations has been studied (Ref 8, 9), and it was concluded that to theoretically calculate the strain-hardening exponent and the strength coefficient as precisely as possible, the existing equations could not be directly used.

In the present article, the equations derived in Ref 8 and 9 are taken as a starting point, and new expressions of both equations are deduced, with the results compared to mechanical test data (Ref 10-12).

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2. Traditional Equations: Background

Traditionally, the relationship correlating the strain-hardening exponent and the strength coefficient with the yield stress-strain behavior is (Ref 4):

$$\sigma_{0.2} = k(0.002)^n \qquad \text{(Eq 1)}$$

These two performance parameters are related to fracture strength and fracture ductility as follows (Ref 1-6):

$$\sigma_f = k\varepsilon_f^n \qquad \text{(Eq 2)}$$

Nomenclature	
k	strength coefficient
n	strain-hardening exponent
ε	total strain
Ψ	reduction-in-area
n_t	theoretical strain-hardening exponent
k_t	theoretical strength coefficient
ε_f	fracture ductility
ε_p	plastic strain
σ	stress
σ_b	ultimate tensile strength
σ_f	fracture strength
$\sigma_{0.2}$	yield strength
α	fracture ductility parameter

In Eq 1 and 2, $\sigma_{0.2}$ is the 0.2% offset yield strength, σ_f is the fracture strength, ε_f is the fracture ductility, n is the strain-hardening exponent, k is the strength coefficient, and 0.002 is the plastic strain corresponding to the yield strength at this point. Equations 1 and 2 have been used to theoretically calculate the strain-hardening exponent and the strength coefficient (Ref 1-6). On the other hand, Eq 2 has been used as the basic formula in predicting metal fatigue crack-initiation life by the equivalent stress-amplitude method (Ref 1-5).

As far as Eq 1 is concerned, the applicability of its use has been studied by the authors (Ref 8) for nine alloys. If the yield strengths from experimental test data are taken as true values, while those calculated from Eq 1 are taken as theoretical yield strength values, then almost all theoretical results are smaller than the true ones. In addition, for four of the alloys (Ref 8) the differences between the theoretical results and the true ones are <10%. However, for the other five alloys, the differences are >10%, with the maximum deviation between them equal to 23%. Therefore, Eq 1 does not accurately describe the relationships among the strain-hardening exponent, the strength coefficient, and the yield stress-strain behavior for the nine alloys.

Equation 2, when applied to the same nine alloys, shows (Ref 9) that some 'theoretical" fracture strengths as derived from these equations are smaller than the test ones, while other 'theoretical" fracture strengths are greater. The differences between the theoretical results and the corresponding experimental test data show for four alloys a deviation <10%. For the other five alloys, the differences are >10% with the maximum deviation equal to 21%. So, Eq 2 also does not properly express the relationships among the strain-hardening exponent, the strength coefficient, the fracture strength, and the fracture ductility for the nine alloys. In addition, when Eq 2 is used to predict fatigue crack-initiation life through the equivalent stress amplitude method, for some alloys the predicted results are close to those of the test data, while for the other alloys the predicted results deviate greatly (Ref 1-6). There may be many reasons for these deviations; however, the intrinsic limitation of Eq 2 may be the important factor.

To highlight the limitations of Eq 1 and 2, the strain-hardening exponent and the strength coefficient are calculated:

$$n = \frac{\log\left(\dfrac{\sigma_f}{\sigma_{0.2}}\right)}{\log(500\varepsilon_f)} \quad \text{(Eq 3)}$$

$$k = 500^n \sigma_{0.2} \quad \text{(Eq 4)}$$

In Ref 10 to 12, the performance parameters for 12 alloys based on experiments have been determined. These results are compared with the values calculated from Eq 3 and 4 as n' and k', respectively, with their precision relative to the test data found in Tables 1 and 2. In these tables, $\delta n' = n' - n/n$ and $\delta k' = k' - k/k$, and the units of $\sigma_{0.2}$, σ_f, k, and k′ are all reported in megapascals. The results listed in Tables 1 and 2, except those for Lc9cgs3 and 40Cr-Mn-Si-Mo-VA, show that the strain-hardening exponents calculated from Eq 3 deviate from the test data by 12.5% to 39.7%. That is to say, the results from Eq 3 are unacceptable. Therefore, Eq 3 cannot be used with any reliability to calculate the strain-hardening exponents. As for the corresponding strength coefficients, although some of the theoretically calculated results when using Eq 4 are close to the test data, they too are suspect because they are derived from the unacceptable calculated strain-hardening exponents.

Table 1 Test data of aluminum alloys (Ref 10-12) and theoretical results

Material	Ly12cz (rod)	Lc4cs	2024-T4	7075-T6	Ly12cz (plate)	Lc9cgs3
ψ, %	16.5	16.6	35	33	26.6	21.0
σ_f	643	711	634	745	618	748
ε_f, %	18	18	43	41	30	28
σ_b	545	614	476	579	476	560
$\sigma_{0.2}$	400	571	303	469	332	518
α, %	3.0	3.0	15.1	13.5	8.0	6.0
n	0.158	0.063	0.200	0.113	0.089	0.071
k	850	775	807	827	545	725
n_t	0.152	0.059	0.190	0.113	0.088	0.066
δn_t, %	−3.8	−6.3	−5.0	0.0	−0.8	−7.0
k_t	835	786	744	824	479	752
δk_t, %	−1.7	1.4	−7.8	−0.4	−12.2	3.8
n'	0.106	0.049	0.137	0.087	0.124	0.074
$\delta n'$, %	−32.9	−22.2	−31.5	−23.1	39.7	4.2
k'	772	774	710	805	717	821
$\delta k'$, %	−9.2	−0.1	−12.0	−2.7	31.6	13.3

As mentioned previously, Eq 1 and 2 originated from the Hollomon equation (Ref 7):

$$\sigma = k\varepsilon_p^n \quad \text{(Eq 5)}$$

In Eq 5, σ is the tensile stress and ε_p is the plastic strain. Equation 5 is a fitted equation using tensile test data (σ and ε). Deviation problems are inevitable when an attempt is made to use the equation at specific points (e.g., $\sigma_{0.2}$, 0.002, σ_f, and ε_f). This may be the main reason why Eq 1 and 2 do not properly correlate the strain-hardening exponent and the strength coefficient to the yield stress-strain behavior, the fracture strength, and the fracture ductility of a wide range of alloys.

3. Theoretical Calculation of Strain-Hardening Exponent and Strength Coefficient

Because Eq 1 and 2 do not properly describe the entire relationship between material performance parameters, new relationships must be found (Ref 8, 9):

$$\sigma_{0.2}^{5/3} = \sigma_b^{2/3} k (0.002)^n \quad \text{(Eq 6)}$$

$$\sigma_f = k\varepsilon_f^n \quad \text{(Eq 7)}$$

for $\alpha < 5\%$ or $10\% < \alpha < 20\%$; and,

$$\sigma_{0.2}^{3/2} = \sigma_b^{1/2} k (0.002)^n \quad \text{(Eq 8)}$$

$$\sigma_f = \frac{\sigma_b}{\sigma_{0.2}} k\varepsilon_f^n \quad \text{(Eq 9)}$$

for $5\% < \alpha < 10\%$, or $\alpha > 20\%$.

In Eq 6 to 9, σ_b is ultimate tensile strength, ψ is the reduction of area, and α is a new fracture-ductility parameter. Its definition is (Ref 8, 9):

$$\alpha = \varepsilon_f \psi = -\psi \ln(1 - \psi) \quad \text{(Eq 10)}$$

Table 2 Test data of alloy steels (Ref 10-12) and theoretical results

Material	30CrMnSiA	30CrMnSiNi2A	40CrMnSiMoVA	08	40	40CrNiMo
ψ, %	53.6	52.3	43.7	80.0	64.0	57.0
σ_f	1795	2601	3512	848	1330	1655
ε_f, %	77	74	63	160	102	84
σ_b	1177	1655	1875	345	931	1241
$\sigma_{0.2}$	1104	1308	1513	262	883	1172
α, %	41.4	38.7	27.7	128.0	65.0	48.0
n	0.063	0.091	0.147	0.160	0.060	0.066
k	1476	2355	3150	531	1172	1579
n_t	0.076	0.096	0.110	0.160	0.060	0.052
δn_t, %	20.6	5.5	−25.0	0.0	0.0	−21.2
k_t	1718	7116	2980	597	1260	1577
δk_t, %	16.4	−10.2	−5.4	9.0	−5.3	−0.1
n'	0.082	0.116	0.146	0.180	0.070	0.057
$\delta n'$, %	30.2	27.7	−0.4	12.5	16.7	−13.6
k'	1839	2694	3754	802	1364	1670
$\delta k'$, %	24.6	14.4	19.2	51.0	16.4	5.8

Comparing Eq 6 to 9 with Eq 1 and 2 shows the following: Eq 6 has an additional factor $(\sigma_b/\sigma_{0.2})^{2/3}$; Eq 8 has a factor $(\sigma_b/\sigma_{0.2})^{1/2}$; and Eq 9 has a factor $(\sigma_b/\sigma_{0.2})$. The appearance of these factors, relative to Eq 1 and 2, relates more appropriately the strain-hardening exponent and the strength coefficient with the yield stress-strain, the fracture strength, and the fracture ductility in Eq 6 to 9 (Ref 8, 9).

Consequently, expressions for both the strain-hardening exponent and the strength coefficient can be obtained:

$$n = \frac{\log\left(\dfrac{\sigma_f^3\sigma_b^2}{\sigma_{0.2}^5}\right)}{3\log(500\varepsilon_f)} \quad \text{(Eq 11)}$$

$$k = \sigma_f \varepsilon_f^{-n} \quad \text{(Eq 12)}$$

for $\alpha < 5\%$ or $10\% < \alpha < 20\%$; and

$$n = \frac{\log\left(\dfrac{\sigma_f^2}{\sigma_{0.2}\sigma_b}\right)}{2\log(500\varepsilon_f)} \quad \text{(Eq 13)}$$

$$k = \frac{\sigma_f\sigma_{0.2}}{\sigma_b}\varepsilon_f^{-n} \quad \text{(Eq 14)}$$

for $5\% < \alpha < 10\%$ or $\alpha > 20\%$.

To examine the utility of this approach, test data for 12 alloys were collected (Ref 10-12), and are listed in Tables 1 and 2. In these tables, the strain-hardening exponents and the strength coefficients calculated from Eq 11 to 14 are denoted as n_t and k_t, respectively. Moreover, in these Tables 1 and 2, $\delta n_t = n_t - n/n$ and $\delta k_t = k_t - k/k$. The units of σ_b, k, and k_t are all reported in megapascals.

Results show that for the aluminum alloys in Table 1, the theoretical strain-hardening exponents and theoretical strength coefficients are much closer to the actual test data. For the alloy steels in Table 2, the theoretical results are not so well-behaved as those for the aluminum alloys. For example, for 40Cr-Mn-Si-Mo-VA the theoretical strain-hardening exponent deviates from the test data by 25%. The difference between the theoretical results and the test data (Ref 10-12) may arise because Eq 11 to 14 are deduced from Eq 6 to 9, while, as has been shown previously (Ref 8, 9), Eq 6 to 9 only approximately describe the relationships among the related performance parameters. Thus, the strain-hardening exponent and the strength coefficient calculated from Eq 11 to 14 must deviate from those in the experiment. Second, while the strain-hardening exponent is the slope of the $\log\sigma - \log\varepsilon_p$ curve, the strength coefficient is the coordinate of the intersection point between the $\log\sigma - \log\varepsilon_p$ curve and the σ axis. However, the $\sigma - \log\varepsilon_p$ curve is a curve fitted to the experimental data. The precision of the curve strongly relies on both the number of tensile stress-strain data points and the degree of scatter between them. So, deviations are inevitable during the fitting process either as a result of the slope calculation or the location of the coordinate of the intersection point.

Traditionally, the strain-hardening exponent and the strength coefficient have been determined by experiment. During the determination process, many tensile tests are performed. The plastic strain ε_p must be measured, and the $\log\sigma - \log\varepsilon_p$ curve was determined. From this curve, the slope of the line and the coordinate of the intersection point between the extrapolated line and the σ axis must be determined. Only after these tasks have been performed are the strain-hardening exponent and the strength coefficient known. However, Eq 11 to 14 shows that if the yield strength, the ultimate tensile strength, the fracture strength, and the fracture ductility are known, then the strain-hardening exponent and the strength coefficient can be calculated. Because these four material performance parameters can be easily determined during one tensile test, the approach using Eq 11 to 14 is quicker and simpler. In addition, the theoretical results from Eq 11 to 14 can be used to quickly examine the correctness of the tensile test data.

4. Conclusion

When tensile test data are lacking, a simple theoretical method of calculating the strain-hardening exponent and the strength coefficient has been suggested. The equations used in the method are:

$$n = \frac{\log\left(\dfrac{\sigma_f^3\sigma_b^2}{\sigma_{0.2}^5}\right)}{3\log(500\varepsilon_f)} \quad \text{and} \quad k = \sigma_f \varepsilon_f^{-n}$$

for $\alpha < 5\%$ or $10\% < \alpha < 20\%$; and

$$n = \frac{\log\left(\dfrac{\sigma_f^2}{\sigma_{0.2}\sigma_b}\right)}{2\log(500\varepsilon_f)} \qquad \text{and} \qquad k = \frac{\sigma_f \sigma_{0.2}}{\sigma_b}\,\varepsilon_f^{-n}$$

for $5\% < \alpha < 10\%$ or $\alpha > 20\%$.

The method is simple and quick to use. It provides results that are as good or better than those using the traditional approach (i.e., using Eq 3 and 4).

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